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# The interlayer exchange coupling in magnetic multilayers: the effect of the thickness of the ferromagnetic layer

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**Abstract.** In this work the effect of the ferromagnetic layer thickness on the interlayer exchange coupling in magnetic multilayers, which has rarely been considered so far, is investigated by employing the one-band tight-binding hole-confinement model. The numerical calculations for a simple cubic lattice show that although the oscillatory variation of coupling parameter  $J$  with the variation of non-magnetic layer thickness  $N$  has periods insensitive to the ferromagnetic layer thickness  $M$ , when  $M$  becomes smaller the amplitude and phase of such oscillations depend strongly on  $M$ , and  $J$  varies with  $M$  in an oscillatory-like fashion. Our theoretical results may be helpful in understanding the recent experimental data.

## 1. Introduction

During recent years, much attention has been paid to the interlayer exchange coupling in magnetic multilayers consisting of alternating magnetic and non-magnetic materials [1–14]. In a number of magnetic multilayers (including the sandwich as a particular case), it has been found experimentally that the interlayer coupling parameter  $J$  is an oscillatory function of the non-magnetic layer thickness  $N$  [3–5] with unexpectedly long period or two periods. A broad spectrum of theoretical approaches has been adopted to explain these peculiar phenomena, such as the first-principles method [6], the tight-binding total-energy calculation [7], the RKKY theory [8, 9], the tight-binding hole-confinement model [1], the free-electron model [10], etc. Great progress has been made towards understanding the origin of the oscillatory behaviour. It is clear now that the *aliasing* effect [11–13], i.e. the fact that  $J$  is sampled at discrete values of  $N$ , can shift the RKKY-like short-period oscillations to long-period ones, and, the two-period oscillations are attributed to the special structure of the Fermi surface.

It seems to be popularly accepted that the interlayer coupling is only a surface effect, insensitive to  $M$ . However, a few experiments with small  $M$  (about 5–10 monolayers), showed [2] that the effect of ferromagnetic layer thickness should not be negligible. Recently, Barnás [10] studied this effect theoretically, using a free-electron model. In our opinion, however, such a model may be not realistic for magnetic multilayers. In a more recent theoretical work [14], the effect of  $M$  was investigated by means of first-principles calculations for  $M = 1, 3$  and  $N = 1, 3, 5, 7$  monolayers. The results showed that the function  $J(N)$  for  $M = 1$  monolayer is significantly different from that for  $M = 3$  monolayers. Obviously, the first-principles calculation is difficult to carry out for larger  $M$  and  $N$ . In this paper, the effect of  $M$  on  $J$  is investigated by numerical calculations based on

the one-band tight-binding hole-confinement model. The system considered is a sandwich consisting of two identical ferromagnetic metallic layers separated by one non-magnetic metallic layer with the same simple cubic lattice structure.

## 2. Model and improvements

The Hamiltonian in the one-band tight-binding hole-confinement model is [1]

$$H = \sum_{(i,j),\sigma} t_{ij} a_{i\sigma}^+ a_{j\sigma} + \sum_i U_i n_{i\uparrow} n_{i\downarrow} \quad (1)$$

where  $a_{i\sigma}$  ( $a_{i\sigma}^+$ ) is the operator annihilating (creating) an electron of spin  $\sigma$  on site  $i$  and  $n_{i\sigma} = a_{i\sigma}^+ a_{i\sigma}$ . The hopping parameters  $t_{ij}$  are assumed to be the same in both metals and  $U_i = \infty$  or 0 in the ferromagnetic or non-magnetic layer, respectively. The Hartree-Fock approach is exact for Hamiltonian (1) in this case [1]. Let the layers be perpendicular to the  $z$  axis.  $H$  can be divided into two terms:

$$H = H(\uparrow) + H(\downarrow) \quad (2)$$

where  $H(\uparrow)$  ( $H(\downarrow)$ ) is the Hamiltonian of electrons with spin  $\uparrow$  ( $\downarrow$ ). In the case of ferromagnetic coupling between two ferromagnetic layers, we have

$$H(\uparrow) = h_{\uparrow}(2M + N) \quad H(\downarrow) = h_{\downarrow}(N) \quad (3)$$

where

$$h_{\sigma}(N) = \sum_{(i,j)} t_{ij} a_{i\sigma}^+ a_{j\sigma} \quad (4)$$

is the Hamiltonian of electrons that have spin  $\sigma$  in the non-magnetic layer consisting of  $N$  monolayers, while in the case of antiferromagnetic coupling, (3) becomes

$$H(\uparrow) = h_{\uparrow}(M + N) \quad H(\downarrow) = h_{\downarrow}(M + N). \quad (5)$$

Now, the original problem has been converted into the simpler one in which only  $h_{\sigma}(N)$  is concerned. In the following, we take  $t_{ij} = t$  ( $t < 0$ ) for  $i, j$  nearest neighbours and  $t_{ij} = 0$  otherwise; the temperature is assumed to be zero ( $T = 0$  K), and the energies are measured in unit of  $2|t|$ . The eigen-energies of  $h_{\sigma}(N)$  are

$$\epsilon(k_x, k_y, r) = -\cos[r\pi/(N+1)] - \cos(k_x a) - \cos(k_y a) \quad r = 1, 2, \dots, N. \quad (6)$$

The number of electrons that occupy levels (6) up to the Fermi energy  $E_F$  is

$$n_e(E_F, N) = \sum_{r,k_x,k_y} \theta[E_F - \epsilon(k_x, k_y, r)]. \quad (7)$$

The total energy of these electrons is given by

$$E(E_F, N) = \sum_{r,k_x,k_y} \epsilon(k_x, k_y, r) \theta[E_F - \epsilon(k_x, k_y, r)] \quad (8)$$

and the corresponding thermodynamic potential is

$$\Omega(E_F, N) = E(E_F, N) - E_F n_e(E_F, N). \quad (9)$$

When  $M \rightarrow \infty$ , the Fermi energy of the sandwich is equal to the bulk Fermi energy  $\tilde{E}_F$  independent of whether the interlayer coupling is ferromagnetic or antiferromagnetic. Thus, the interlayer coupling parameter can be expressed as

$$J = [\Omega_{\text{FM}}(\tilde{E}_F) - \Omega_{\text{AF}}(\tilde{E}_F)]/S \quad (10)$$

where  $S$  is the area of the film, while  $\Omega_{\text{FM}}(E_F)$  and  $\Omega_{\text{AF}}(E_F)$  are the thermodynamic potentials for the ferromagnetic and antiferromagnetic couplings, respectively. Although (10) has been extensively utilized, it is no longer useful in the present work because in our case  $M$  is small and hence the variation of Fermi energy ( $\sim O(1/M)$ ) is not negligible. The significant effect of  $M$  on  $J$  in the case of small  $M$  is in part due to the variation of  $E_F$  with  $M$ . Equations (2.2)–(2.12) of [1] are suitable only in the large- $M$  limit. Hence in the following calculations we use the formula

$$J = [E^{(\text{FM})}(E_F^{(\text{FM})}) - E^{(\text{AF})}(E_F^{(\text{AF})})]/S. \quad (11)$$

Here,

$$E^{(\text{FM})}(E_F^{(\text{FM})}) = E(E_F^{(\text{FM})}, 2M + N) + E(E_F^{(\text{FM})}, N) \quad (12)$$

and

$$E^{(\text{AF})}(E_F^{(\text{AF})}) = 2E(E_F^{(\text{AF})}, 2M + 2N) \quad (13)$$

are the total energies of electrons related to the ferromagnetic and antiferromagnetic interlayer coupling, respectively, and  $E_F^{(\text{FM})}$  and  $E_F^{(\text{AF})}$  stand for the corresponding Fermi energies determined by

$$n_e(E_F^{(\text{FM})}, N) + n_e(E_F^{(\text{FM})}, 2M + N) = N_e \quad (14)$$

and

$$2n_e(E_F^{(\text{AF})}, M + N) = N_e \quad (15)$$

respectively, where  $N_e$  is the total number of electrons in the sandwich. The relation between  $N_e$  and the bulk Fermi energy  $\tilde{E}_F$  is given by

$$N_e = 2(M + N) \lim_{L \rightarrow \infty} [n_e(\tilde{E}_F, L)/L]. \quad (16)$$

The bulk Fermi energy  $\tilde{E}_F$  is the only adjustable parameter in our calculations. The summation over  $k_x, k_y$  in equations (7) and (8) can be converted into an integral with respect to energy  $\epsilon = -\cos(k_x a) - \cos(k_y a)$ :

$$n_e(E_F, N) = \sum_r \int d\epsilon N_{2D}(\epsilon) \theta \left[ E_F - \epsilon + \cos\left(\frac{r\pi}{N+1}\right) \right] \quad (17)$$

$$E(E_F, N) = \sum_r \int d\epsilon N_{2D}(\epsilon) \left[ \epsilon - \cos\left(\frac{r\pi}{N+1}\right) \right] \theta \left[ E_F - \epsilon + \cos\left(\frac{r\pi}{N+1}\right) \right] \quad (18)$$

where  $N_{2D}(\epsilon)$  denotes the two-dimensional density of states, for  $-2 < \epsilon < 0$ ,

$$N_{2D}(\epsilon) = -\frac{S}{\pi^2 a^2} \int_1^{-\epsilon-1} \frac{dx}{\sqrt{(1-x^2)[1-(\epsilon+x)^2]}}. \quad (19)$$

Therefore, for  $-3 < E_F < -1$ , we have

$$n_e(E_F, N) = \frac{S}{\pi^2 a^2} \sum_{r=1}^{j_F} \tilde{n}_e \left[ E_F + \cos \left( \frac{r\pi}{N+1} \right) \right] \quad (20)$$

$$E(E_F, N) = -\frac{S}{\pi^2 a^2} \sum_{r=1}^{j_F} \left\{ 4\tilde{E} \left[ E_F + \cos \left( \frac{r\pi}{N+1} \right) \right] + \cos \left( \frac{r\pi}{N+1} \right) \tilde{n}_e \left[ E_F + \cos \left( \frac{r\pi}{N+1} \right) \right] \right\} \quad (21)$$

where

$$j_F = \min \left\{ N, \text{int} \left[ \frac{N+1}{\pi} \cos^{-1}(-E_F - 2) \right] \right\} \quad (22)$$

$$\tilde{n}_e(\epsilon) = \int_0^{\cos^{-1}(-\epsilon-1)} \cos^{-1}(-\epsilon - \cos x) dx \quad (23)$$

$$\tilde{E}(\epsilon) = \int_0^{1+\epsilon/2} \ln \left[ \frac{4x^2 + 4 - \epsilon^2 + \sqrt{(4x^2 + 4 - \epsilon^2)^2 - 64x^2}}{8x} \right] dx. \quad (24)$$

**Table 1.** The variation of  $\lambda$  (the oscillation period of  $J$  as a function of  $N$ ), in monolayers with  $\tilde{E}_F$  (in  $2|t|$ ) when  $M = 5$  monolayers

$\tilde{E}_F$	-1.05	-1.1	-1.2	-1.3	-1.4	-1.5	-1.6	-1.7	-1.8	-1.9
$\lambda$	11	7	5	4	4	3	3-2	3-2	3-2	2
$\tilde{E}_F$	-2.0	-2.1	-2.2	-2.3	-2.4	-2.5	-2.6	-2.7	-2.8	-2.9
$\lambda$	2	2	2	2-3	3-2	3	3-4	4	5	7

### 3. Results and discussion

In table 1, the calculated oscillation period  $\lambda$  of  $J(N)$  versus the relative bulk Fermi energy  $\tilde{E}_F$  is given for  $M = 5$ . Here and in the following the thicknesses, such as  $M$ ,  $N$ , and  $\lambda$ , are all in the units of monolayers. This result agrees qualitatively with  $\lambda \sim \pi / \cos^{-1} |2 + \tilde{E}_F|$  of [1] for  $M = \infty$ , implying that the period of  $J(N)$  is insensitive to  $M$ . However, when  $M$  is small, the phase and amplitude of  $J(N)$  are clearly  $M$  dependent. The variations of  $J$  with  $N$  corresponding to  $M = 5, 10$ , and  $200$  obtained by us are shown in figure 1, for  $\tilde{E}_F = -1.05, -2.9$ , and  $-2.6$ . Our results show that by fixing  $N = N_0$ ,  $J(M, N_0)$  behaves like damping oscillations of  $M$  around  $J(\infty, N_0)$ . Independent of whether the coupling  $J(\infty, N_0)$  is ferromagnetic or antiferromagnetic, the interlayer coupling  $J(M, N_0)$

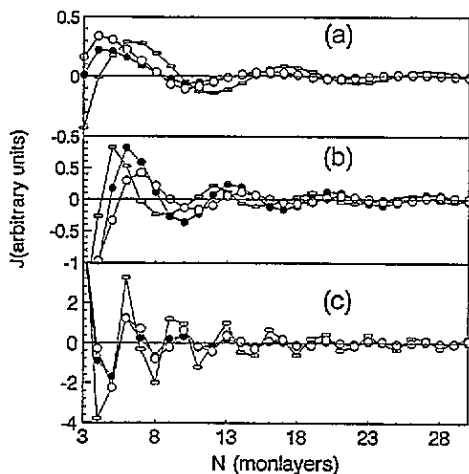


Figure 1. Exchange couplings  $J(N)$ :  $M = 200$  (open circles), 10 (solid circles) and 5 (rectangles). The lines are guides for the eye. (a)  $E_F = -1.05$ ; (b)  $E_F = -2.9$ ; (c)  $E_F = -2.6$ .

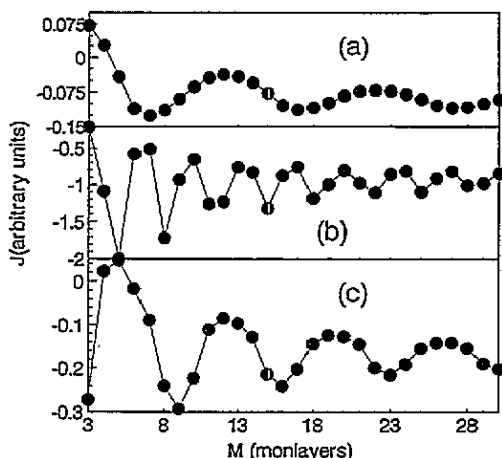


Figure 2. The exchange coupling  $J$  as a function of  $M$ . The line is only a guide for the eye. (a)  $E_F = -1.05$ ,  $N = 10$ ; (b)  $E_F = -2.6$ ,  $N = 8$ ; (c)  $E_F = -2.9$ ,  $N = 11$ .

Table 2.  $J_1$  and  $J_2$  (the first and second peaks of  $J$  as a function of  $N$ , in arbitrary units) as well as  $N_1$  and  $N_2$  (the thicknesses of the non-magnetic layer corresponding to  $J_1$  and  $J_2$ , respectively, in monolayers) for different  $\tilde{E}_F$  (in  $2|\epsilon|$ ).

		$M$						
		4	5	6	7	8	9	10
$\tilde{E}_F = -1.2$	$J_1$	-1.3	-0.7	-0.9	-1.5	-1.8	-1.3	-1
	$J_2$	-0.4	-0.1	-0.2	-0.5	-0.6	-0.3	-0.2
	$N_1$	5	5	5	5	5	5	5
	$N_2$	9	10	10	10	10	10	10
$\tilde{E}_F = -2.8$	$J_1$	-0.5	-0.3	-1.1	-1.3	-0.8	-0.5	-0.5
	$J_2$	-0.14	-0.08	-0.5	-0.6	-0.4	-0.1	-0.1
	$N_1$	6	7	7	7	6	7	7
	$N_2$	11	12	12	12	11	12	12
$\tilde{E}_F = -2.9$	$J_1$	-0.56	-0.24	-0.11	-0.09	-0.24	-0.34	-0.36
	$J_2$	-0.23	-0.11	-0.03	-0.02	-0.10	-0.14	-0.16
	$N_1$	9	8	9	10	11	10	10
	$N_2$	16	15	16	18	18	17	17

may be ferromagnetic, antiferromagnetic, or even zero depending on the thickness of the ferromagnetic layer, provided that  $J(\infty, N_0)$  is not far from zero. An example of our results is illustrated in figure 2. It can be seen that the oscillation period of  $J(M)$  with  $N$  fixed is roughly the same as that of  $J(N)$  with  $M$  fixed. In table 2, the dependences of the first and second peak values,  $J_1$  and  $J_2$  respectively, of  $J(N)$  on  $M$  are given for  $\tilde{E}_F = -1.2$ ,  $-2.8$ , and  $-2.9$ . Here,  $N_1$  and  $N_2$  are the thicknesses of the non-magnetic layer corresponding to  $J_1$  and  $J_2$ , respectively. These results show once more that the amplitude of  $J(N)$  is sensitive to  $M$ . The recent experimental data [2] manifested that  $J$  oscillates with  $M$ , which can be considered as supporting our theoretical results.

#### 4. Summary

Based on the one-band tight-binding hole-confinement model [1] in which it is assumed that the exchange split is large enough in ferromagnetic layers, we have investigated the effect of the ferromagnetic layer thickness  $M$  on the interlayer exchange coupling  $J$  in a magnetic sandwich for a simple cubic lattice structure at  $T = 0$ . Numerical results show that when  $M$  is small (about 5–10 monolayers) the phase and amplitude of oscillations of  $J$  as a function of  $N$  are sensitive to  $M$ , but not so for the oscillation period. In addition,  $J$  varies in an oscillatory-like fashion with  $M$  and the oscillation period of  $J$  as a function of  $M$  with  $N$  fixed is roughly the same as that of  $N$  with  $M$  fixed. Some recent experimental data support qualitatively our theoretical results.

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